

# Neural Correlates of Gender, Culture, and Race and Implications to Embodied Thinking in Mathematics

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**Abstract** In this chapter, I discuss neuroscience research and selected findings that are relevant to mathematics education. What does it mean, for example, to engage in a neuroscientific analysis of symbol reference? I also discuss various research programs in neuroscience that have useful implications in mathematics education research. Further, I provide samples of studies conducted within and outside mathematics education that provide a neural grounding of gender, culture, and race. The chapter closes with three brief implications of neuroscientific work in mathematics education research, in general, and in individual- and intentional-embodied cognition in mathematical thinking and learning, in particular.

## 1 Introduction

If one has to hazard a guess, the science of the early 21st century will be driven by brain research. . . . At the core of the new brain science is an astounding mix of technologies adapted from other sciences. None has been developed with the brain in mind, but they have radically transformed neuroscience. (Hacking 2004, p. 26)

Functional neuroimaging techniques pick up on signals indicating brain activity. These signals, by themselves, do not specify a behavior. Only by linking these brain signals with behavior do they have psychological meaning. (Phelps and Thomas 2003, p. 755)

Broadly speaking, the aim of Cognitive Neuroscience is to elucidate how the brain enables the mind. . . . to constrain cognitive, psychological theories with neuroscientific data, thereby shaping such theories to be more biologically plausible. (Ansari et al. 2011, p. 1)

While there have been significant advances in neuroscientific methods, tools and techniques, and findings in the last decade, neuroscience, a term coined in the 1960s, as a scientific field that studies brain structure and functioning is still in its emergent state. Hacking (2004) points out that current interests and investments in neu-

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rosience in fields outside of education<sup>1</sup> basically seek the resolution of medical issues such as the negative effects of aging and related diseases and disabilities that we all will experience at some point in our lives. Certainly, there is as well a “narcissistic” expectation that “as we learn more about the brain we shall learn more about human nature, about ourselves and our kind” (Hacking 2004, p. 27), especially “our own mental processes” and “higher cognitive functions” (Editorial 2003, p. 1239) in both individual and collective contexts, which could be potentially controversial in many cases.

Despite and amidst healthy skepticism and productive critique in various theoretical and methodological components of neuroscience research in nonmedical contexts (e.g., Anderson and Reid 2009; Coch and Ansari 2009; Fuson 2009; Geertz 2000; Hardcastle and Stewart 2002; Kaufmann 2008; Willingham 2009), I share the view of Ansari et al. (2011) in the opening epigraph, that the impressive eruptive findings in this field will consequently provide valuable descriptive (versus prescriptive) information—cautionary tales, perhaps (Goswami 2005)—about neural mechanisms that support cognitive processes in mathematical thinking and learning. ‘Descriptive’ means that the neurally drawn information is not meant to be interpreted as a recipe manual for optimal learning but as providing knowledge or a level of explanation that sees mathematical thinking as also being about mind/brain functioning and relationships (Anderson and Reid 2009; Ansari 2005). Unfortunately, current mathematics education research knowledge, practice, and policy appear to dawdle through scientific endeavors that address the material or biological components in both cognitive and affective analyses relevant to, say, the learning of concepts, skills, and other processes (Campbell 2006; Grabner et al. 2010; Schlöglmann 2003).

The title to this chapter involves the frequently used term “neural correlates” in matters relevant to neuroscientific data to convey the current content of available empirical evidence. Various findings drawn from neuroscientific experiments in both nonmathematical and mathematical contexts offer, at least for the time being, supporting evidence that is primarily correlational (versus causal) in nature. Correlations between a target behavior and an activated region in the brain do not mean the latter is involved in the former. “[T]here is some relationship,” Phelps

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<sup>1</sup>Certainly motives behind interests in neuroscience outside education depend on stakeholder contexts. Hacking (2004) articulated medical interests as an example. Neuroscientific findings and programs in the in nonmedical issues that bear on national security (National Research Council 2008) are also of interest to federal and military agencies in the USA. In 2008, the NRC published the document, *Emerging Cognitive Neurosciences and Related Technologies*, in which an attempt is made to address ways in which neuroscientific knowledge could be used to eventually develop usable “future warfighting applications” (p. 14) for the intelligence community. Such applications would have neuroscience associated with the following tasks: (1) “read” the “cognitive states and intentions of persons of interest;” (2) “enhance” the “cognitive capacities” of soldiers (how to make them learn faster and process information more quickly and precisely than usual, how to help them make correct decisions when engaged in battle); (3) “control” the “states and intentions” of oneself (e.g., pain, fear) and others (e.g., “disrupt” an “enemy’s motivation to fight”), and: (4) “drive devices” via “cognitive states” (e.g., using white noise to impair senses, using neuropsychopharmacology to develop drugs that “target specific sensory receptors”) (pp. 16–17).

and Thomas (2003) note, but “an activation response does not inform us as to what, exactly, a brain region does in the generation of a behavior” (p. 753). Nevertheless, the correlational findings should help “deepen our understanding of causal mechanisms” (Goswami 2009, p. 176) underlying, say, skills and representations involved when learners engage in mathematical knowledge construction. Also, consistent and strong correlations may be used to infer neural markers (or what Goswami 2009, calls biomarkers), neural signatures, or neural specificities (Cantlon et al. 2009) with more data that could then provide useful information in constructing relevant and more reliable cognitive measures, assessments, and diagnostic tools.

My interest in neuroscience, in particular those studies that directly tackle issues relevant to mathematical cognition, has been spurred by Thagard’s (2010) thoughts about the role of neural processes in making sense of embodied thinking. Embodied action is an emerging area of research interest among mathematics educators around the globe. Two instances of this kind of work involve understanding functions of representational gestures in conveying mathematical meaning, for example, the special *Educational Studies in Mathematics (ESM)* issue on gestures and multimodality in mathematical contexts (Radford et al. 2009), and the influence of emotions and other affective factors in sustaining interest in mathematical knowledge construction, for example, the special *ESM* issue on affect in mathematics education (Zan et al. 2006). Thagard (2010) points out, and rightly so, the mutually determining relationships between neural and psychological accounts of human actions, that is, that our “cognitive capacities” could be seen as a complex of “representational/computational abilities that outstrip embodied action” (p. 9). The purported gendered/cultural/racialized nature of surprise, insight, perception, abduction, creativity, emotion, inference making, meaning construction and signifying practices, and so on, that all bear on mathematical thinking processes might “be illuminated by consideration of neural mechanisms” (Thagard 2010, p. 449). Current models of mathematical thinking are, in fact, based on representations that are both physically available (e.g., gestures) and linguistic, which can also be analyzed in computational and neuroscientific terms.

This chapter is organized in five sections. Section 2 clarifies the different (but overlapping) contexts and purposes of neuroscience that are pertinent to issues in mathematics education. What does it mean, for example, to engage in a neuroscientific analysis of symbol reference? Also briefly discussed are various research programs in neuroscience that have useful implications in mathematics education research. Section 3 provides selective examples of work in which a neuroscientific analysis in understanding basic mathematical processes was employed. This section is meant to showcase recent exciting investigations in which attempts are made to ground, albeit not entirely, mathematical thinking in neurophysiological terms. I also point out constraints and limitations of such investigations so that a cautionary habit is developed of seeing where the science ends and the speculation begins, a disposition that Bruer (1999) would have readers acquire, given the allure of developing misleading brain-based educational implications (e.g., mixing correlation and causation; overgeneralizing) or “neuromyths” (Organization for Economic Cooperation and Development [OECD] 2007) on very limited and targeted experiments.

Section 4 provides samples of studies within and outside mathematics education that provide a neural grounding of gender, culture, and race. Section 5 includes three brief implications of neuroscientific work in mathematics education research in general, and in individual- and intentional-embodied cognition in mathematical thinking and learning in particular.

## 2 Analyzing Issues in Mathematics Education from a Neuroscience Perspective

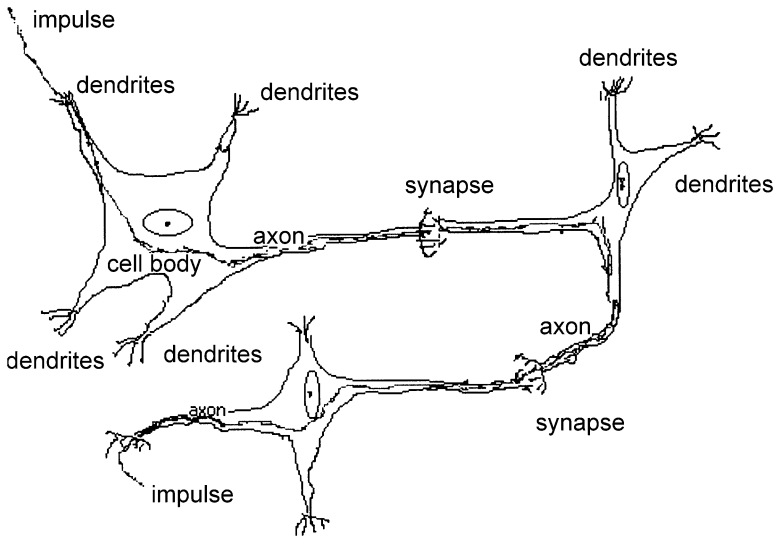
Modules in the brain or distributed patchworks? (Ansari 2008, p. 279)

Basically, neuroscience involves studies of brain functioning and development or, more generally, the human nervous system (Szűcs and Goswami 2007). One implication of this characterization for educational research involves situating the brain in a mediating function so that changes and developments in psychological or behavioral processes can be explained in material terms by understanding the constraints and connections that emerge when brain cells interact with one another (see Fig. 1; Ansari et al. 2011; Pennington et al. 2007). The connections influence either individual structures or pathways between two or more structures. Thus, brain development is seen to be driving developmental changes in various aspects of behavior in both individual and collective contexts. Acquired experiences also exert influence in brain development. Neuroscientists aptly refer to this as synaptic plasticity (Howard-Jones 2008), and learning is one purposeful tool in which experiences are acquired. In school mathematical contexts, for example, children learn mathematical concepts and processes through “targeted experiences” (Szűcs and Goswami 2007, p. 114) of the exact nature, which come into contact with their “approximate number sense” that has been neurologically established to be a characteristic of both human and animal brains. In nonschool mathematical contexts such as the home, the manner in which young children experience being cared for early in their development is correlated with adult behavior. From a neuroscientific perspective this means that their experiences help produce brain cells that affect their memory functions and how they cope with stress in the long term (Eliot 2010).

Learning is a central issue in any study involving educational phenomena. In neuroscience, concerns about learning are routed through studies involving memory, that is, sensory<sup>2</sup>, short term, long term, and working memory (Howard-Jones

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<sup>2</sup>Sensory memory lasts for a few seconds and quickly keeps and discards copies of immediately acquired visual and auditory information. Short-term memory (STM) is a short-term storage of information transferred from sensory memory and does not manipulate the acquired information. STM provides a space for engaging in quick calculations and holds visual and auditory information. Working memory (WM) is the active operational component in STM. It actively processes information acquired in STM and is central in the development of language, reading, mathematics, and problem solving. WM also deals with attentional resources in STM such as the ability to concentrate on one aspect of a target object and shutting off others. Long-term memory (LTM) stores information over periods of time and is organized via schemes that join together to form



**Fig. 1** Brain cell networking (reprinted with permission from [http://www.duboislc.org/EducationWatch/JCameron/01\\_09\\_02\\_HowWeLearn.html](http://www.duboislc.org/EducationWatch/JCameron/01_09_02_HowWeLearn.html))

2008). How the human brain functions and neurally correlates with various parts of the body, and the physical and linguistic operations that come with acting and thinking are all indications of a neuroscientific approach to the study of learning. Aside from learning, the development of one’s identity as a gendered, cultured, and racialized being is also a significant educational issue. In this chapter I focus on issues surrounding learning and identity; other concerns in education such as curriculum, instruction, and assessment are not discussed. Certainly, any discussion surrounding equity issues in education involves understanding how these five elements can be aligned well together; this is not of concern here. But I should point out that neuroscientific findings in learning and identity could be used to develop useful implications in the effective design and delivery of curriculum, instruction, and assessment. Space prohibits an exploration of all these aspects in full detail, but I recommend readers to access the most recent references on this matter which are provided in the bibliography.

Goswami (2004) notes that understanding learning at the neuroscientific level involves determining ways in which synapses (i.e. junctions between nerve cells) work in neural (or neuronal) functioning (see Fig. 1). The human nervous system consists of neurons or cells that process and transmit information via synaptic signaling. This signaling occurs in a structure that enables neurons to connect to each other eventually forming a network. The nervous system also includes the brain, the

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new knowledge structures. Readers are referred to Menon (2010) for an extended discussion of the neuroanatomical correlates of working memory and other relevant cognitive processes relevant in the development of mathematical thinking and skills.

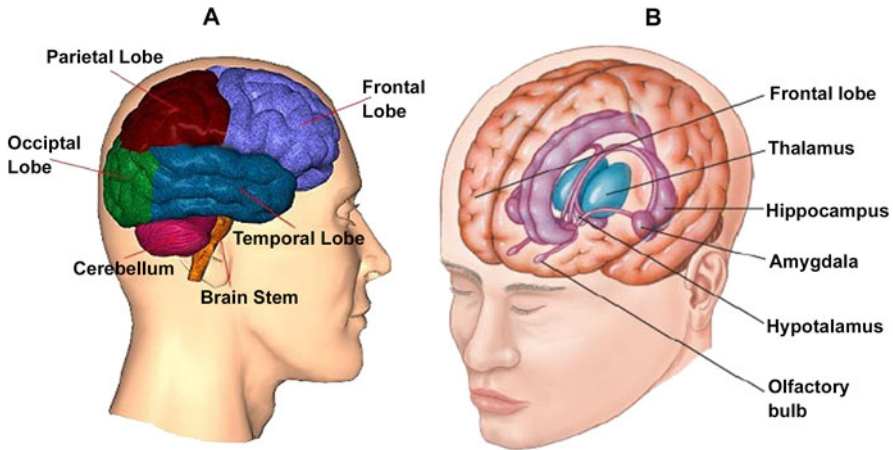
spinal cord, and the peripheral ganglia. Ganglia are tissue masses of bundled nerve cells; they mediate and serve as relay points between two or more neuronal systems such as the peripheral (nerves) and central nervous (brain and spinal cord) systems. Neurons come in several types depending on function. There are sensory neurons that react to sound, touch, and light, and which affect the sensory organ. There are also motor neurons that affect muscle movement, and neural patterns exist that are associated with specific mental representations or mental states. Thus, from a neuroscientific perspective, (successful) learning constitutes (effectively) understanding how changes occur in the neural connections in an individual or among individuals in a social context. Bechtel's (2002) thoughts concerning the complementary analytic approach needed in neuroscientific work echoes the following important point initially raised by Petersen and Fiez (1993) about being wary of the difference between brain localization of a task (i.e. the neuromyth of brain modules) and brain localization of an information processing operation:

[E]lementary operations, defined on the basis of information processing analyses of task performance, are localized in different regions of the brain. Because many such elementary operations are involved in any cognitive task, a set of distributed functional areas must be orchestrated in the performance of even simple cognitive tasks. ... A functional area of the brain is not a task area; there is no "tennis forehand area" to be discovered. Likewise, no area of the brain is devoted to a very complex function; "attention" or "language" is not localized in a particular Brodmann area or lobe. Any task or "function" utilizes a complex and distributed set of brain areas. (Petersen and Fiez 1993, p. 513)

Figure 2 illustrates the different major subdivisions of the cerebral cortex in a human brain. The cerebral cortex consists of two mirror halves that are often referred to in terms of the left and the right sides of the brain (i.e. brain laterality). The hippocampus and the amygdala are located in the midbrain, under the cerebral cortex. The cortex itself is the largest part of the brain that deals with higher brain functioning such as thinking and acting. The hippocampus is associated with long-term memory and affects spatial orientation performance. Four lobes or sections divide the cerebral cortex and engage in different activities, as follows:

- The *frontal lobe* is engaged in activities that involve planning, reasoning, problem solving, and controlling speech, movement, and emotions;
- The *temporal lobe* is engaged in activities that pertain to memory, language, and recognition of auditory stimulus;
- The *parietal lobe* is engaged in activities that involve the use of spatial processing, orientation, perception, recognition, movement, and touch;
- The *occipital lobe* is engaged in activities that tap vision and visual processing.

A simple example of a neuroscientific investigation is instructive at this stage. Neuroscience research has produced an interesting finding concerning gender differences in spatial processing that is important because it is correlated with mathematical ability. Males and females have been neuroscientifically assessed to employ different neural patterns when thrown in an unfamiliar environment, with the males' left hippocampus showing increased activation compared to females' (Editorial 2005; Grön et al. 2000). Of significance here are sample size and numerical



**Fig. 2** The human cerebral cortex and the major lobes (reprinted with permission from: (A) <http://www.neuroskills.com/edu/ceufunction1.shtml>; (B) <http://cwx.prenhall.com/bookbind/pubbooks/morris5/chapter2.html>)

amounts of differences in neuroscientific experiments; these values need to be considered in context and analyzed based on the particular tasks presented to the participants. Small differences, for instance, may have neuroscientific value but carry little to no educational impact. Even when there are differences, they may only make sense relative to the tasks used in the study (Bruer 1999). Of concern is the temptation to infer causes on a single part (or parts) of the brain (e.g., the very misleading implications of brain modules and laterality localizations) when the available neuroscientific evidence is basically concerned with establishing correlations. As Szűcs and Goswami (2007) have noted, like Petersen and Fiez (1993) before them, “no complex representation can be localized in a single part of the brain” and that “complex phenomena are coded by the interplay of various interconnected neural networks” (p. 115; cf. NRC 2008). Further, we need to know when neuroscientific results end and psychological explanations begin. For example, the observable finding that females in unfamiliar settings employ landmarks while their male counterparts use Euclidean properties of space (e.g., land shape, distances between walls in a room) is a psychological observation. It is, however, reasonable to assume that there is a mutually determining relationship between particular neural and psychological functions<sup>3</sup>.

<sup>3</sup>In particular, it is worth noting the interesting methodological reflections of Poldrack (2006) and Henson (2006) concerning ways functional neuroimaging data are employed in developing arguments that they term as *forward* and *reverse inferences*, which involve establishing relationships between cognitive functioning and brain activation; for ethical issues involving neural-based reverse inferences, see Poldrack 2008. Forward inferences are deductively valid, and proceeds from assessing neural activity on the basis of performing certain cognitive tasks. Reverse inferences are deductively invalid since they involve making conclusions about cognitive functioning on the basis of brain activation. For Poldrack (2006),

To measure neural activity in the brain, blood flows are actually monitored. A positron emission tomography is not used in educational neuroscientific contexts since it is highly invasive and requires injecting participants with radioactive tracers. Instead, functional magnetic resonance imaging (fMRI), which emerged in 1991, is used to measure neural activity throughout the brain in a non-invasive manner. Participants, oftentimes 5-year-old children and older<sup>4</sup>, in recent fMRI experiments wear a head cap that allows a brain scanner to monitor and measure changes in blood flows in the relevant brain regions. What happens is that increased neural activity activates a demand for oxygen, which is then delivered to neurons by hemoglobin. Consequently, increased blood flows occur in the appropriate brain regions. Differences in the magnetic resonance signals of blood (oxygenation) are then used to detect brain activity (i.e. what is oftentimes referred to as the “blood oxygenation level dependent” imaging technique). There are other safe, noninvasive, and indirect neuroimaging techniques (e.g., multiple electrode recording that measures the interactions of brain cells in different cortical lobes, and near-infrared spectroscopy that scans cortical tissue and measures changes in blood hemoglobin concentration) and those that are used to overcome weaknesses (such as time delay) in an fMRI machine (e.g., using a multimodal technique of fMRI and (scalp-based) electroencephalography, which yields gamma and alpha rhythm data waves that measure traces and electrical signals drawn from activation of neurons), but they are not central to the discussion in this chapter (see Bandettini 2009 for an impressive methodological reflection of fMRI technology in neuroscientific work). Suffice it to say, there are indirect ways of measuring neural processes and patterns underlying cognitive activity. Neural patterns that represent sequences of neural firings are indications that some information is being transmitted in a particular way. For example, the 18 adults in an fMRI study by Delazer et al. (2003) have been shown to initially activate their frontal cortical areas associated with working memory in performing long multiplication. With continued practice, activation then shifted to the parietal areas (including shifts within the parietal areas) which are associated

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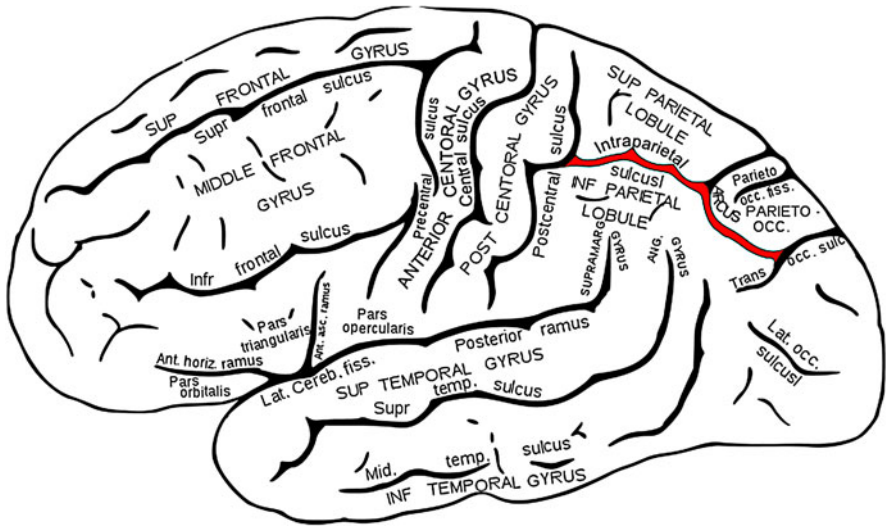
cognitive neuroscience is generally interested in a mechanistic understanding of the neural processes that support cognition rather than the formulation of deductive laws. To this end, reverse inference might be useful in the discovery of interesting new facts about the underlying mechanisms. Indeed, philosophers have argued that this kind of reasoning (termed ‘abductive inference’ by Peirce), is an essential tool for scientific discovery” (p. 60).

However, Henson’s (2006) point below is a reminder about being mindful of neuroscientific claims:

[I]t is important to think carefully about the type of inferences that can be made from functional neuroimaging data... only by making these caveats and assumptions explicit, and criticizing them, will we be able to assess the real value of functional neuroimaging for cognitive science” (p. 68).

<sup>4</sup>Kaufmann (2008) points out that current fMRI experiments are restricted to 5-year-old children and older because “fMRI technique requires participants to be awake and respond to stimuli presented in the (narrow and very noisy) scanner environment while simultaneously task-processing related changes in the blood oxygen consumption in different brain regions are recorded” (pp. 2–3).





**Fig. 3** Lateral surface of the left cerebral hemisphere ([http://en.wikipedia.org/wiki/Intraparietal\\_sulcus](http://en.wikipedia.org/wiki/Intraparietal_sulcus))

with automatic processing (Fig. 3). Thus, neural processing of long multiplication in skilled individuals over time tends to shift from quantity-based processing to automatic retrieval.

There are at least three different kinds of neuroscience research programs. While there are clear overlaps insofar as methodology, tools, and analyses, knowing them by label helps identify their primary and fundamental points of interest.

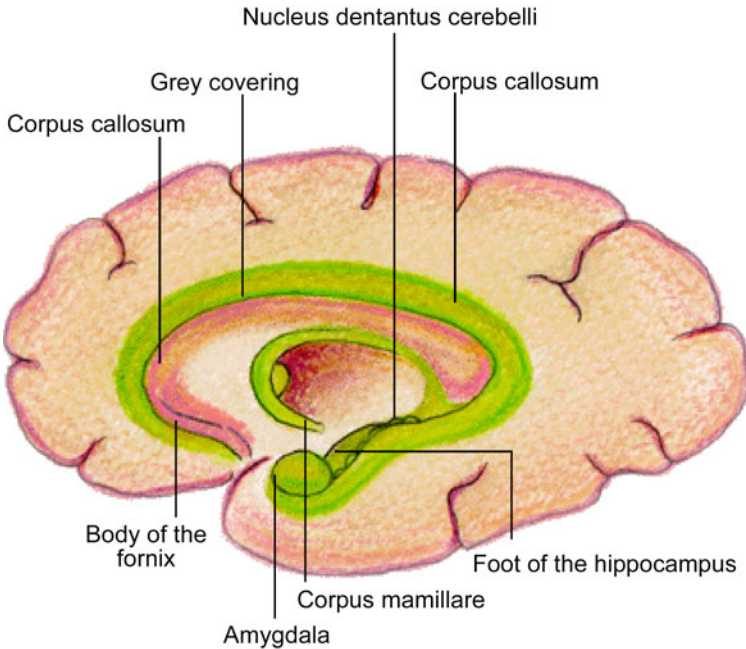
*Cognitive neuroscience* emerged in the mid-1980s due to dissatisfaction with cognitive psychological theories and findings in scientifically and empirically resolving foundational issues in cognition such as the content of mental images (Pennington et al. 2007). For example, initial studies in neuroscience on the perceptual versus linguistic content of mental images have materially established a neural basis favoring perceptual processing. The methodology in the related cognitive neuroscience experiments adopted connectionist modeling and electrochemical activity, both of which try to give an account of mental states in terms of networks of neural patterns that give rise to learning and development (Goswami 2008). *Developmental cognitive neuroscience* emerged at about the same time. It is basically interested in establishing neural markers of development among individuals relative to some symbol system (e.g., language). While cognitive neuroscience is fundamentally concerned with brain-behavior relations in young and older children, developmental cognitive neuroscience is interested in understanding how such relations evolve over time. Going against the static “hardwired-at-birth” view of cognition, this research program empirically established the existence of developing neural circuits, which then led to the view of mental models as resulting from probabilistic epigenesis and neural constructivism. Here the psychological theories of Piaget have been given a material basis in terms of neural patterns and processes. Further progress in connec-

tionism and molecular genetics has spawned studies in typical and atypical developments and developmental neurobiology (the ontogenetic emergence of the nervous system) and evolutionary developmental biology (the evolutionary emergence of different forms of species).

*Social/cultural cognitive affective neuroscience* is also a recent field of research interest among (initially) developmental and (later) social and cultural psychologists. Social neuroscience began in 1992, the year that marked the beginning of the Decade of the Brain (Adolphs 2010). Initially, it was concerned with the “neurobiology of social behavior” (Adolphs 2010, p. 752). Currently, it involves establishing neural correlates of social cognition, higher-order processes such as moral reasoning, social coordination and cooperation, and perceptions such as stereotyping and prejudice (about race, social status, etc.). Research concerns in cultural neuroscience, Chiao (2011) points out, are

motivated by two intriguing questions about human nature: How does culture (that is, values, beliefs, practices) shape neurobiology and behavior, and how do neurobiological mechanisms (that is, genetic and neural processes) facilitate the emergence and transmission of culture? (Chiao 2011, p. 240)

There is no longer any interest in comparing brain sizes and shapes for the purpose of ascertaining biological primacy and permanent differences between and among different cultures and race (Eberhardt 2005; Todorov et al. 2006). In this field, the fundamental aim is “to understand human diversity” (Ames and Fiske 2010, p. 78) that is “perhaps our most precious ability” (Chiao 2011, p. 247). For example, neuroscientific techniques were used in investigating the powerful social phenomenon called *theory of mind* in young children in developmental psychology experiments. The findings indicated that children at about the age of 4 years already understand the intent and mental states of those others around them. Todorov et al. (2006) underscore the complex and intertwined neural connections between affect and cognition, which explains why they introduced the term social/cultural cognitive affective neuroscience as a research field that “entails cognition, emotion, motivation, and readiness for behavior” (p. 82). For example, brain-imaging experiments have shown greater activity in the amygdala and insular lobe (located between the frontal and temporal lobes, which deals with emotions and interpersonal experiences—see Fig. 4) in both black and white participants when they were shown black and white faces. However, the activation decreased over time with more exposure to the colored faces. A study by Sanfey, Rilling, Aronson, Nystrom, and Cohen (2003) had a group of 19 strangers initially meeting with an experimenter prior to a scanning session. The activity involved evaluating their emotional reactions on both fair and unfair proposals in a simple game that involved two players (with one assigned the proposer, the other the responder) split a sum of money. Sanfey et al. (2003) also saw significant neural activity in the anterior insular lobe. Apparently, the participants manifested their disgust neurally (and behaviorally) via a strongly activated anterior insular lobe in situations when a proposer made an unfair proposal. However, the neural response was not strong in situations when a computer program was used to make



**Fig. 4** Amygdala and the insular lobe (reprinted with permission from <http://www.hpssandiego.com/VB73a2.jpg>)

the same unfair proposal. In a constrained fMRI environment, social neuroscientists basically create psychologically meaningful situations (Todorov et al. 2006, p. 77) that symbolize particular social situations in order to assess participants' neural responses on tasks presented to them in an activity, which may then be correlated with, and could predict, an expected (and perhaps unexpected) behavioral response.

*Educational neuroscience* is a term recently introduced by Geake (2005), which refers to programs that use educational, neuroscientific, and cognitive psychological methods in understanding mental representational structures. Such structures pertain to how the brain codes information through electrochemical activity, and structural changes could be materially analyzed in terms of cortical changes that occur in individuals. Where a neuroscience approach could complement typical approaches used, cognitive psychology involves establishing a relationship between neural and symbolic activity. Thus, educational neuroscience offers a unifying framework that brings together the analysis of high-level descriptions of the mind (symbolic representations, psychological theories) and lower level data and theories (neuronal activity and function); education is seen as developing "optimal ways of shaping and enriching" individual learners' cognitive systems and mental representations (Szűcs and Goswami 2007; Goswami 2008, 2004). Once again, the intent is not about identifying or localizing particular mental functions in the brain, since the functions develop and emerge on the basis of dynamic and distributed neural and

cortical networks and pathways that change over time with more experiences, and are recruited to fulfill other relevant cognitive functions. Szűcs and Goswami (2007) clearly articulate one central finding of neuroscience research that should caution educators about lateral recommendations in mathematics education theory and practice, as follows:

The idea that there is no all-knowing, inner central executive that governs what is known and that orchestrates cognitive development is very important for education. It means that education must deal with the “vast parallel coalition of more-or-less influential forces whose... unfolding makes each of us the thinking beings that we are” (Clark 2006, p. 373). (Szűcs and Goswami 2007, p. 116)

Mental representation of numbers, for example, involves a coordinated activity that taps the parietal lobe, which codes our “approximate sense” of magnitude, and the angular gyrus (Fig. 3) and language-processing parts of the brain, which store memorized arithmetical facts.

In closing this section, I briefly discuss Nieder’s (2009) research investigation concerning the neurobiological evolution of symbolic thinking and reasoning in humans and nonhuman primates, which demonstrates the usefulness of neuroscientific findings in developing a case for a material or neural approach to understanding possible biomarkers of abstract thinking in humans. Further details are provided in the next section in which selective findings from the neuroscience of mathematical cognition are presented; for now it simply makes sense to say that humans’ evolved number sense ability actually represents a symbol system that does not merely reflect “isolated sign-object associations” (Nieder 2009, p. 99). Some animals have been shown to be capable of limited numerical competence, but it is for the most part nonsymbolic. This means that they operate only at the indexical level (i.e. associations). For example, monkeys and pigeons can be trained to perform simple single-digit and approximate addition, and subtraction limited to very small cardinalities. In humans, number sense competence transcends the indexical status of such associations to include the ability to understand the necessary abstract relationships between signs and the objects they represent, made possible by language. Where neuroscientific methods are useful is in laying neural foundations that support behavioral or psychological explanations and, thus, allows an understanding of how humans and nonhuman primates produce (semantic) meanings to (object) symbols. For example, on the basis of current neurobiological studies with animals and humans, it appears that the dorso-lateral granular prefrontal lobe may be neurally responsible for the indexical skill (e.g., macaque monkeys performing shape-to-numerical value associations). The prefrontal lobe is the anterior part (forehead side) of the frontal lobe (see Fig. 3) that does not affect movement when electronically stimulated; one of its psychological roles involves executive function, which deals with processes that pertain to abstract thinking and rule development. The neural correlates of symbolic competence in humans over time (i.e. childhood to adolescence to adulthood) can be initially explained via prefrontal lobe activation. This then shifts to parietal lobe and temporal lobe activations in later development as competence in language improves.

### 3 Neural Correlates of Mathematical Concepts and Processes

A central question in cognitive science is whether natural language provides combinatorial operations that are essential to diverse domains of thought. . . . We find that only linguistic reasoning excites the known neural substrate of language comprehension, whereas algebra recruits bilateral parietal regions previously implicated in number and magnitude representation. This double dissociation suggests that, at least in the mature brain, the manipulation of algebraic expressions does not rest on the neural machinery of natural language. (Monti et al. 2011, p. 1)

In the psychology of mathematics education research, studies involving the triad of gender, culture, and race (and, more generally, equity) are pursued in terms of their relationships to mathematical learning and thinking. Hence, in this section, the neuroscientific ramp of mathematical cognition is skidded to keep in mind ways in which such findings might inform, change, and advance recent and emerging mathematics-education related theories involving the equity triad. Due space constraints, I engage in a selective reading of exemplar work in the neuroscience of mathematics cognition that addresses the following three basic themes relevant to a study of mathematical thinking and learning in schools, namely: (1) mathematical processing, which is a central skill; (2) linguistic processing, which is a basis for exact mathematical understanding, and; (3) visuospatial processing, which is necessary in visual and spatial (geometric, diagrammatic) thinking. Readers are referred to the special *ZDM* issue on cognitive neuroscience and mathematics learning (Grabner et al. 2010) for further details, but I note one important point raised by Obersteiner et al. (2010) whose work appears in the special issue addressed to the mathematics education community in particular. I echo their view that mathematics education-driven neuroscientific research investigations have the potential to increase the relevance and translational dimension of current cognitive neuroscientific results in mathematical cognition beyond findings drawn from basic numerical tasks presented to participants. Expertise and experience decisions to be made about which complex tasks to use and subject to a meaningful analysis on the basis of their relevance to school mathematical practices and content. Further, since factors such as age, task characteristics, and mathematical competence are often taken into serious consideration in experiments, it is possible to produce a more relevant psychological-neural account of mathematical thinking and learning.

#### 3.1 *Mathematical Processing*

There has been a tremendous amount of research activity in the area that deals with the neural correlates of arithmetical thinking and skills (e.g., Delazer et al. 2003; Fehr et al. 2007; Kadosh et al. 2008; Rosenberg-Lee et al. 2009; Zamarian et al. 2009; Rocha et al. 2005). Drawn primarily from the influential work of Dehaene (1997, 1992), there is strong and converging neural evidence from various neuroscientific experiments across several countries in different contexts that indicate “number and arithmetic” as being “more than cultural conventions and may

have their ultimate roots in brain evolution” (Dehaene et al. 2003, p. 487). Meta-analytic results show that there are regions in human (and animals’) brains that support nonverbal or language-independent mental magnitude processing, which enable engagement in simple and complex arithmetical comparison and operation tasks<sup>5</sup>. Dehaene’s (1992, 1997) neural network model of triple coding shows that humans possess: (1) an auditory verbal code (left temporal lobe), which is used to recall automatized arithmetical facts such as the multiplication table; (2) a visual code (occipital lobe) for the Hindu-Arabic notation, which is used to perform arithmetical and symbolic operations and make parity decisions, and; (3) a language-independent analog magnitude code (parietal lobe), the famous “mental number line”, that allows comparisons to be made among very small numbers, as well as engagement in approximate arithmetic and other spatial judgments of relative sizes. Approximate number processing<sup>6</sup> has been neurally documented to occur *not* in any one particular region but in the parietal, prefrontal, and cingulated regions of the brain with the horizontal segment of the bilateral intraparietal sulcus (“HIPS”; see Fig. 3) as being primarily responsible for the representation and manipulation of numerical quantities and the other regions fulfilling a supportive function in working memory (Dehaene et al. 2004; also see Ansari 2008; Varga et al. 2010 for extended discussions of the HIPS). Jacob and Nieder (2009) also found the same activation pattern network in the case of fraction representation, that is, the “fronto-parietal cortex is tuned to preferred fractions, generalizing across the format of representation” (p. 4652).

Delazer et al. (2003) note the effects of training in significantly modifying patterns in neural activity (e.g., shifts from frontal to parietal activation, or from the intraparietal sulcus to the angular gyrus, or from quantity-based processing to automatic retrieval networks). Dehaene et al. (2004) also point out that arithmetical tasks such as counting and multiplying compared to addition and subtraction might show greater activation in the other regions of the brain, perhaps because they rely on language-based fact retrieval systems or rote verbal memory. For more updated

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<sup>5</sup>See Varga et al. (2010) and Nieder (2005) for syntheses of research comparing human and animal competence involving concepts of counting, cardinality (numerical quantity), and order (rank) from neuroscientific and neurobiological perspectives. For fMRI comparisons between children and adults involving different aspects of arithmetical processing, see Rocha et al. (2005) and Kawashima et al. (2004). See Ansari (2009) for a review analysis of results drawn from various neuroscientific studies that focused on developmental disorders and difficulties involving numerical cognition and relevant mathematical processes.

<sup>6</sup>Current interests in the implications of approximate number sense are linked to its possible reverse-inferential relationship to school mathematics achievement for both children and adults. For example, based on a longitudinal assessment of 64 14-year-old children with normal development that started in kindergarten, Halberda et al. (2008) established a strong correlation between individual children’s approximate number sense and their past scores on standardized school mathematics achievement tests. Mazocco et al. (in press) established a strong correlation between domain-specific deficits in approximate number processing and persistently deficient mathematics achievement among children with mathematical learning disabilities (i.e. those who scored below the 10th percentile in a mathematics achievement test).

findings, see: Fehr et al. (2007) about the role of working memory capacity and relevant neural networks in complex arithmetical calculations; and Rosenberg-Lee et al. (2009) about different cortical activations that employ the same neural regions due to differing applications of arithmetical strategies such as multiplying from left to right versus right to left and different ordered count-on strategies for adding whole numbers.

Recently, a few studies have focused on other mathematical content beyond arithmetic such as algebra (e.g., Anderson et al. 2003; Lee et al. 2007; Monti et al. 2011; Sohn et al. 2004; Terao et al. 2004). Lee et al. (2007) assessed 18 participants' (all right-handed; 10 males and 8 females whose ages ranged from 20 to 25 years) problem-solving strategies involving typical algebra word problems. Lee et al. (2007) compared and contrasted, at the neural level, the impact of schematic diagrams and alphanumeric symbols in algebra problem solving in working memory. Constructing diagrams taps visual processing while constructing algebraic expressions relies on relevant numerical processing. Results of their fMRI study indicate that no extensive neural differences were found that favored the use of one method over the other, however, there were differences in terms of differential engagement within similar neural processes. The HIPS was actively engaged in both diagrammatic and variable conditions, perhaps as a result of having to compare magnitudes. But what appears to be a more interesting result deals with the neurally drawn finding that using alphanumeric representations actually demands more working memory resources than using diagrams to solve algebra word problems. Monti et al. (2011) neurally assessed 21 right-handed healthy adults when they reasoned about the equivalence and grammatical well-formedness of pairs of linguistic (e.g. "x gave y to z" and "z was given y by x") and algebraic (e.g. "y is greater than z divided by x" and "x times y is greater than z") statements. Results of their fMRI experiment indicate that linguistic equivalence primarily recruited the left fronto-temporal perisylvian (linguistic) regions, while "algebraic equivalence evoked no more activity in these regions than is necessary for simple reading... [but] recruited areas previously reported for number cognition," the bilateral portions of the HIPS (Monti et al. 2011, p. 3).

Developing and assessing inductive and deductive arguments play a central role in constructing and understanding proofs and, more generally, in developing sophisticated mathematical understanding. While there is converging neuroscientific evidence showing activation of the bilateral fronto-parietal network regions in arithmetical (and algebraic) processing, several recent studies in which the neural correlates of human reasoning were investigated have demonstrated empirically the dominance of the left hemisphere in adult participants who were tested on problems involving inductive and deductive arguments (Goel and Dolan 2004; Goel et al. 1997, 1998, 2000). Both inductive and deductive items activated the left prefrontal lobe. Further, while not evident in deductive processing, inductive processing significantly activated the left dorsolateral prefrontal cortex, and this may be "due to the use of world knowledge in the generation and evaluation of hypotheses" (Goel and Dolan 2004, p. B120). Also, while not evident in inductive processing, deductive processing significantly activated the linguistic neural network and Broca's Area on

the basis of significant activation of the left fronto-temporal regions, indicating engagement in syntactical processing (due to the logical form of the propositions) and the use of working memory resources.

### ***3.2 Linguistic Processing***

Linguistic processing has been neurally observed to activate the left hemispheric brain regions regardless of ability or disability. What is worth noting is the emerging body of neuroscientific evidence that indicates the neural structures of mathematical cognition as not being linguistic-mediated (or notation-mediated) despite being mediated by numeric symbols (Butterworth et al. 2008; Cantlon et al. 2006; Monti et al. 2011), a view that runs contrary to the Vygotskian thesis that language is necessary for thinking (Brannon 2005), including the well-accepted view that “language forms the basis of structured thought across cognitive domains” (Monti et al. 2011, p. 6). Supporting behavioral evidence has been observed among aphasic (i.e. language-impaired) individuals who have been found incapable of processing simple and complex grammatical relationships such as “The man killed the lion,” “The lion killed the man,” and “This is the dog that worried the cat that ate the rat that ate the malt that lay in the house that Jack built” but capable of manipulating simple and complex numerical expressions such as  $52 - 11$ ,  $11 - 52$ , and  $(3 + 17) \times 3$  (Brannon 2005, p. 3177; Varley et al. 2005, p. 3519). Certainly, computational recursion is a principle common to both language and mathematics. However, neuroscientific evidence shows that each tends to operate and function on its own, following a distinct syntactical structure. There is also recent work that neurally links the activation of the frontal and parietal lobes (see Fig. 1) in cases when numerical tasks involve the use of linguistic quantifier terms (e.g., some, all, most, more, at least; cf. Hubbard et al. 2008).

### ***3.3 Visuospatial Processing***

A recent review of research studies by de Hevia, Vallar, and Girelli (2008) on the role of visuospatial processing in various aspects that matter to arithmetical computing presents converging evidence indicating a “close relationship between numerical abilities and visuospatial processes” (p. 1361). As noted in the previous section on mathematical processing, individuals appear to rely on a spatial mental number line in performing approximate arithmetical tasks. de Hevia et al. (2008) also noted activations of the visuospatial working memory distributed structures, including various regions in the parietal lobe and the supramarginal and angular gyri (see Fig. 3). Interests relevant to possible cultural differences in arithmetical processing have consequently highlighted the salience of visual processing in this domain. For example, a recent brain imaging study by Tang et al. (2006) has shown that



neural arithmetical processing appears to be influenced by cultural practices. When the thinking processes of 12 native Western adult participants and 12 native Chinese university students were compared in relation to a simple visually-presented arithmetical task asking them to determine whether a third digit was greater than the bigger one of the first two in a triplet of Arabic numbers, they actually found cortical differences in the way the two groups engaged in number processing. In this particular task, the Chinese participants processed visually, while the Western participants processed verbally, indicated by the increased activation of their left perisylvian lobe language region. Tang et al. (2006) then hypothesized that the visual dominance in number processing among the Chinese participants, evidenced by the activation of their visuo-premotor regions, could be psychologically explained by their reading experiences in school which involves repeatedly learning Chinese characters, and their early experiences in using an Abacus which activated the production of mental images that are visual forms.

## 4 Neural Correlates of Gender, Culture, and Race

In this section, I provide additional examples of neurocorrelational studies conducted within and outside mathematics education in the areas of gender, culture, and race, with the view that possible neural differences might be used as a scientific basis in designing appropriate and meaningful instruction- and classroom-related strategies or programs that encourage the development of positive findings and reduce the negative effects (e.g., negative emotions, fear, and other stereotyped effects associated with mathematical learning; cf. Hinton et al. 2008; Schlögmann 2003). Fiske (2007) identified that the neural determinants of social behavior in people can be changed over time, that is, “[p]eople will always gravitate toward the familiar and similar, but they can expand their boundaries, if sufficiently motivated. And this is the substance of social science married to neuroscience” (p. 159). It should be noted that the selected sample of studies in this section excludes those that focus on visual correlates of behavior using eye-tracking methodology.

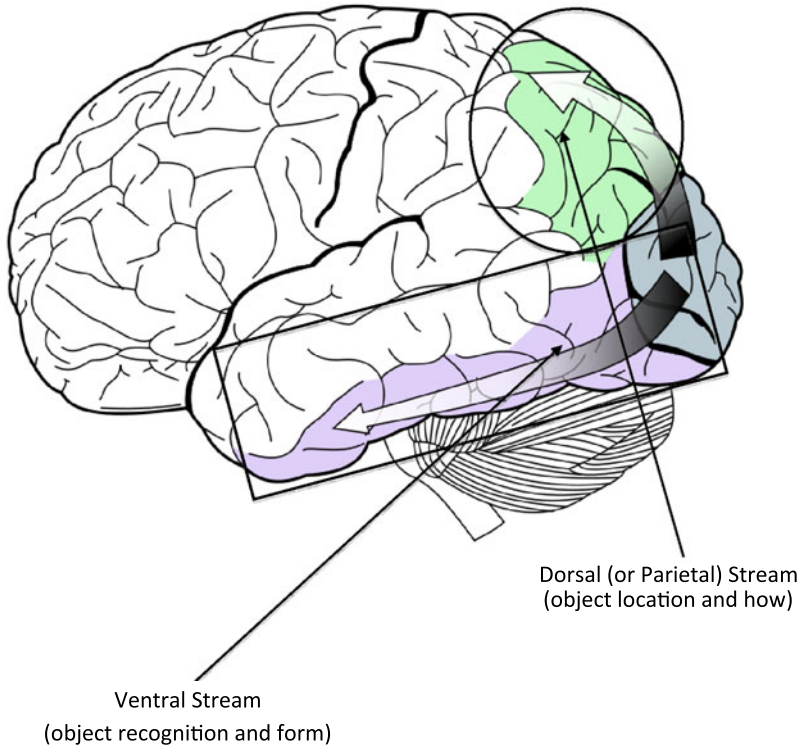
### 4.1 Gender

Documenting possible gender differences in spatial thinking with the help of fMRI technology appears to be a major focus of productivity in research in this area. While most studies have established gender differences at the neural level, many of them have proven to be controversial due to constraints in, and the nature of, the tasks presented to participants (see, Cela-Conde et al. 2009, and Kaiser et al. 2008 for brief surveys of relevant studies). One study, in particular, conducted by Kaiser et al. (2008) illustrates a combined neural-psychological analysis relevant to this field of research. The authors investigated the effect of gender on the neural

correlates of spatial perspective taking in 12 educated, right-handed adult males and 12 adult females. They found that while both groups activated similar neural patterns in solving a presented task, there were gender differences, albeit small, in strategy use. The women consistently used an egocentric (self) perspective approach and the men employed object-based strategies that neurally activated their precuneus and right inferior frontal gyrus (see Fig. 2).

A study by Keller and Menon (2009) provides an example of a neuroanatomical analysis which involves investigating gender differences in cortical activation (brain functioning) and in gray matter density and volume (brain structure). Keller and Menon (2009) assessed whether there were gender differences between 24 male and 25 female (right-handed) educated adults (aged 18 to 36 years) in various aspects of their brain regions and processing on a relatively simple mental arithmetical task that asked them to determine whether presented addition and subtraction equations (three single-digit whole-number terms on one side and the result on the other) were true or false. The mental arithmetical task has already been shown to activate the appropriate neural regions in the brain (see mathematical processing section above), which then allowed the authors to focus on possible gender-related variations. Their results show that, at the functional level, while overlaps were found in the neural substrates between the male and female participants, gender differences surfaced with greater activation in males than in females around the two right hemisphere regions encompassing the dorsal (right IPS and right angular gyrus) and ventral (right parahippocampal gyrus and right lingual gyrus) visuospatial streams (see Fig. 5). These regions are activated whenever tasks involve number, space, and visual information with the ventral stream proceeding to the temporal lobe and the dorsal stream toward the parietal lobe. Further, at the anatomic or structural level, gender differences were found with the female participants having greater gray matter density and volume than the males in those brain regions that showed activation differences. Isolated from any psychological observation, the authors conjectured that male and female adults might be employing different cognitive strategies (e.g., mental versus overt techniques) despite producing similar performance results. Also, the authors used their structural finding in inferring that differences in the amount of gray matter might explain the functional finding concerning differences in cortical activation.

Two recent lines of gender-related research that use functional imaging techniques and which matter to mathematics education establish neural correlates of: (1) visual aesthetic preference and appreciation with additional interpretive analyses drawn from evolutionary models of symbol use, and parietal differences between humans and animals which could also explain the location versus Euclidean approaches on spatial tasks noted in women and men, respectively (Cela-Conde et al. 2009), and (2) empathic ability, which seems to indicate that males and females process emotional tasks differently on the basis of different neural processing strategies, with males activating the cortical and cognitive-related regions and females the amygdala, inferior frontal regions, and emotion-regulated regions (Derntl et al. 2010).



**Fig. 5** Dorsal and ventral streams ([http://en.wikipedia.org/wiki/File:Ventral-dorsal\\_streams.org](http://en.wikipedia.org/wiki/File:Ventral-dorsal_streams.org))

## 4.2 Race

Two recent syntheses of research by Ito and Bartholow (2009) and Phelps and Thomas (2003) on the neural correlates of race in nonmathematical contexts implicate the regions involving the fusiform gyrus and posterior cingulate cortex (see Fig. 3) in race-based face and familiar face perceptions. Individuals significantly activate the region within their fusiform gyrus when shown faces that belong to the same race as they do. This has led to the phenomena of same-race superiority (or same-race advantage) (Phelps and Thomas 2003) and relevant in-group bias. The superiority condition benefits individuals who relate “more naturally” with members that come from the same group. The increased activation might also be explained in terms of an acquired visual expertise that evolves over time and through social practices that encourage individuals to categorize and classify (Van Bavel et al. 2008). Racial categorizing consequently involves activating stereotype beliefs and prejudice that also influence one’s personal and perceptual judgments and attitudes toward others. The distributed neural networks involving the posterior cingulate cortex are activated in familiar face contexts. Identifying a person we know, that is, a familiar face, goes beyond visual familiarity to include person knowledge and emo-

tional response (Gobbini and Haxby 2007). Person knowledge involves the subjective characteristics (personal traits, intentions, attitudes, mental states, and objective information of a familiar person) and recruits the anterior paracingulate cortex, the posterior superior temporal sulcus/temporoparietal junction, and the prucuneus. Emotional responses are tied to activation of the amygdala (tied to attitude towards a person) and the insula (tied to evoked responses towards a face, or faces, that yield an intense emotional effect).

Ito and Bartholow (2009) offer the following bleak implications below concerning how the complexity of race relations and race processing from a neural perspective might be understood:

[A]ttempts to get people to not “see” race will be relatively ineffective. ... [C]hange occurring at the single level of stereotypical or evaluative associations is unlikely to eliminate racially biased behavior because biased responses could still occur through processes mediated by other parts of the neural network. ... [I]nterventions that seek to improve behavior regulation capabilities might be effective in at least reducing the expression of bias. ... [A]lthough race relations will be affected by race-specific beliefs and feelings, the expression of bias will also be determined by an individual’s general regulatory abilities. (Ito and Bartholow 2009, pp. 529–530)

Consequently, the above implications suggest that consideration be given to the possibility of dissociations between intentional and unintentional biases (Phelps and Thomas 2003). For example, several studies have consistently shown that while white Americans explicitly claim they are not biased toward black Americans, implicit measures indicate a negative bias. Further, Phelps and Thomas (2003) recommend “combining the psychological and neural approaches is the best way to advance our understanding of these complex human behaviors more rapidly and with more clarity than could be achieved using either approach in isolation” (p. 754).

### 4.3 Culture

There are at least two major lines of neurocorrelational studies involving self-other and other-other relations that have implications in the development of understanding of social learning in mathematical contexts. Other people’s intentions apart from one’s own play a significant role in knowledge acquisition relevant to institutional knowledge. Culture-driven ways of seeing and processing also influence various aspects of basic cognitive processes such as perception, attention, number, language, etc. Due to space constraints, findings from only two recent studies are highlighted. Readers are referred to the research synthesis of Ames and Fiske (2010) who discuss the neural bases of cultural differences in major aspects of cultural cognition.

Concerning self-other relations, in psychology what is known as the *theory of mind* (TOM) involves understanding how young children and adults come to understand people’s intentions and mental states other than their own. Neural correlates of TOM have produced mixed results. Some studies with adults implicate the bilateral ventro-medial prefrontal lobe and temporo-parietal junction (see Fig. 3), while others see activation in the medial prefrontal lobe and anterior cingulated regions.

Kobayashi et al. (2007) note that the mixed results perhaps “indicate that some of the neural basis of TOM may be universal whereas others may vary depending upon the person’s cultural or linguistic background” (p. 97). However, regardless of cultural and linguistic backgrounds, the “universal” activation of the ventro-medial prefrontal lobe in individuals might be an indication that they are also engaged in reading and conceptualizing other people’s emotional behavior, which is central to TOM processing (Kobayashi et al. 2007). Gilbert and Burgess (2008) draw on several other neuroscientific studies in which the medial (rostral) prefrontal lobe is seen as “a region playing an important role in social cognition” (p. 150), especially its role in making judgments about oneself and others and in reflecting on people’s emotions and mental states and one’s own, the latter being “an important precursor to metacognitive knowledge conducive to efficient learning” (Gilbert and Burgess 2008, p. 150). Concerning other-other relations, neuroscientists have begun to explore possible neural foundations of cultural differences in processing everyday and mathematical phenomena. For example, the fMRI study of Tang et al. (2006; see Visuospatial Processing section) provided neural correlates of Western versus Eastern ways of processing numbers, number relationships, and number operations with a small sample of 12 Western and 12 Eastern native adults. It should also be noted that the two groups activated the occipitoparietal regions, which indicates that some common neural processing across individuals is manifest.

## 5 Implications for Mathematics Education Research

Knowledge about the brain . . . can be relevant in both designing sound educational programs and evaluating existing educational programs, but neuroscience must be considered as just one source of evidence that can contribute to evidence-based practices in education. . . —it should not be considered alone, out of context from theory or behavioral evidence or the classroom. (Coch and Ansari 2009, p. 547)

Usable knowledge from [educational neuroscience] is already making important contributions to the field of education. . . The research brings a powerful capability to directly intervene in children’s biological makeup, stirring ethical questions about the very nature of child rearing, and the role of education in this process. We argue that there is a key distinction between *raising children* and *designing children*, and the ethical application of neuroscience research to education critically depends upon ensuring that we are *raising* children. (Stein et al. 2011, p. 803)

In this concluding section, I briefly note three implications of neuroscience methodology to mathematics education research.

*First*, Anderson and Reid (2009) have pointed out the need to clearly articulate three different levels of explanation in any research study that combines neuroscience and education research, namely, biological, cognitive, and behavioral. As demonstrated in the preceding sections, the biological or neural accounts are descriptive in nature, but cognitive and behavioral accounts tend to be more prescriptive and normative (cf. Christodoulou and Gaab 2009; Willingham 2009). Current methodological conditions are still not capable of using all the levels at the same

time, hence there is work to be done. Where the methods and interests of educational researchers may differ from typical neuroscientific researchers lie in assessing how proposed cognitive theories and models can be used to understand the resulting behavioral and neural outputs that emerge in activity. The algebra research of Lee et al. (2007; see Mathematical Processing section above) provides an exemplar of this three-level approach.

*Second*, any analytic discussion concerning the nature, theory, and practice of embodied cognition in mathematics has to take into account the neural dimension in thinking and learning (cf. Campbell 2006; Schölglmann 2003). Recent characterizations of embodiment tend to dwell on the sensuous or nonrational aspect, such as the role of representational gestures and artifacts in the environment in conveying and developing mathematical meaning. What is not articulated and discussed are the complex neural mechanisms that support and constrain those behaviors within and outside the self who thinks and learns<sup>7</sup>. Concerned stakeholders are, of course, wary of the return of information processing models in mathematical cognition, but this chapter is clear about the psychological-neural or mind-brain nexus. Neuroscience researchers basically establish neural correlates of human behavior and performance on tasks and those involved in mathematics education research possess the requisites and experiences, to borrow Cerulo's (2010) words, "to tell the rest of the story" (p. 120) by situating neuroscientific findings and their implications in issues that concern embodied thinking, acting, and learning in (school) mathematical contexts. In practical terms, it is unlikely that neuroscience will provide usable knowledge for teaching effectively. Hence, the task of mathematics education researchers is to work hard, experiment, and develop and test new design research techniques and hypotheses (Tommerdahl 2010) based on currently available neuroscientific knowledge that is correlational in nature.

From a developmental cognitive neuroscientific perspective, a neural understanding of embodied cognition involves seeing how, say, the frontal-to-parietal lobe mechanism takes place in activity. In such a mechanism, initial learning involves the use of working memory in the frontal lobe. With more training (through learning and experiences), a shift to the parietal lobe occurs as evidenced by automaticity. However, from a social/cultural affective neuroscience perspective, embodied cognition involves seeing how social and cultural (i.e. the triad of race, gender, culture) construction could also be influenced by neural mechanisms that affect emotions, including one's and other people's views (and stereotypes) of the self in relation to others. The main point being addressed here is the complexity of contexts that matter in any account of embodied cognition which should not be limited to psychological or directly observable behaviors. For example, the following neural findings influence embodied thinking: (1) synaptic plasticity; (2) changes in brain tissue development that affect cognitive ability; and (3) symbol-dependent structures that reflect

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<sup>7</sup>See Gentilucci and Corballis (2006) for an interesting and thought-provoking neuroscientific-based account of the evolution of speech and language from manual gestures to vocal communications (i.e. "gestural-origins theory;" an account that differs from the typical sound-to-language perspective).

differing cultural practices which influence neural performance (e.g., visual versus verbal activation in calculating). What is learnt from plasticity in neuroscience is the significant role of experiences in enhancing or constraining, say, cognitive similarities and differences in people. Also learnt is that some personal experiences, which drive those similarities and differences, may not be cultural and linguistic but neural.

Two neurally drawn psychological findings that matter to embodied cognition involve understanding the impact of representations in mathematical learning, as follows: (1) differing mathematical practices place stress and greater demand on working memory if not developed appropriately (e.g., alphanumeric versus visual approaches in solving word problems; right-to-left versus left-to-right—formal mathematical algorithms—approaches in adding and multiplying whole numbers); and (2) levels of representational precision and fluency do not start from the verbal to the symbolic but, especially among young children, from nonverbal and approximate to nonverbal and exact to verbal and counting-based (Clements and Sarama 2007) which have clear implications in the development of algebraic processes such as generalization and abstraction.

What I term *intentionally-embodied cognition* takes as given the crucial role of social and cultural values, beliefs, intentions, and practices in mathematical thinking, learning, and relevant affective processing, which involve both psychological and neurophysiological processes (Chiao et al. 2008). For example, findings drawn from research studies in social cultural affective neuroscience that deal with self-construal style—that is, individualism and collectivism—have much to inform current sociocultural models of embodiment in mathematical processing. Based on converging evidence from several sources, Chiao et al. (2008) note the influence of cultural beliefs on brain-behavior relations involving visual perception and visual experiences. In psychological studies, the notion of culturally preferred style refers to the finding that Caucasian-Americans, trained to live independently, consistently engage in analytic perception (e.g., changes in individual objects independent of context), while East Asians, trained to live interdependently, consistently employ holistic perception (e.g., changes influenced by context).

A modified version of the famous Framed Line Test (FLT) was used in an fMRI study that helped establish, in neural terms, the correlation between visual perception and cultural views of the self. Hedden et al. (2008) recruited twenty adults (10 native European-Americans and 10 East-Asians recently residing in the USA) to participate in a study that measured their blood-oxygenated level-dependent responses using fMRI on a matching FLT experiment. In the experiment, they were asked to judge (easy or difficult) vertical segment lengths in either absolute (ignoring context) or relative (attending to context) conditions. They were initially shown a square frame with a printed segment drawn vertically from the center of the top edge of the square. They were then asked to judge whether a succeeding square frame of a different size had either the same vertical segment shown in the first frame (an absolute task) or a vertical segment whose proportion relative to the size of the succeeding frame reflected the same proportion as the segment-to-frame in the first square (a relative task). In psychological studies that replicated the FLT task

and used other simple visual tasks, culturally preferred styles were noted; Westerners consistently performed better than Easterners on absolute conditions, while the latter consistently performed better than the former on relative conditions. In the fMRI study of Hedden et al. (2008), the fronto-parietal network, which is generally associated with working memory, cognitive control, and attention, and is thus employed in demanding tasks, was activated over the temporo-occipital regions in both groups on culturally nonpreferred judgments, including the left inferior parietal lobe and right precentral gyrus in situations involving culturally preferred judgments. Increased activation of the fronto-parietal regions was observed in all participants on difficult tasks that were judged to be incongruous with their cultural preferred style. Also, East Asians who reported more association with the American culture than their own did not show increased activation on absolute tasks; this shows the influence of experience in reshaping neural components. Hedden et al. (2008) concluded that “the cultural background of an individual and the degree to which the individual endorses cultural values moderate activation in brain networks engaged during even simple visual and attentional tasks” (p. 12). This finding suggests that the effects of cultural interaction in visual processing occur in the fronto-parietal lobe regions rather than the temporo-occipital lobe network that has always been implicated in early- or primary-stage perceptual processing.

The above finding of Hedden et al. (2008) in regard to the fronto-parietal over the temporo-occipital network activation and the critical synthesis offered by Chiao et al. (2008) in relation to their goal of establishing a cultural basis to theories of consciousness allow further enrichment of understandings of the neural correlates of intentional embodied cognition in mathematical learning. *Contra* radical constructivist arguments characterize individuals in an embodied, primary all-knowing cognitive capacity, and as having the “ability to represent one’s own thoughts, feelings, and intentions as distinct from another” (Chiao et al. 2008, p. 65). Intentional embodiment takes as given the role of culture

in shaping the very nature of conscious experience, such as a person’s conceptualization and experience of themselves and their relations to others. That is, an individualist has a psychological experience and neural representation of themselves that is distinct from another whereas a collectivist has a psychological experience and neural representation of self-knowledge that overlaps with others. (Chiao et al. 2008, p. 65)

Due to technological advances, the nature of school mathematical knowledge is slowly experiencing a visual turn. Neural and neuropsychological findings by Hedden et al. (2008) and many others whose results are periodically synthesized (e.g., Ames and Fiske 2010; Chiao et al. 2008; Ito and Bartholow 2009; Phelps and Thomas 2003) further support the significant role of the sociocultural context in shaping experiences and identifications on visual task performances.

*Third*, the dark side of intentional embodied cognition in mathematical thinking and learning involves those neural findings relevant to negative attitudes of individuals toward others. Brain-behavior patterns of prejudice and stereotype will always complicate conversations involving equity in mathematics education, which consequently translate into inequitable sociocultural practices. Hyde and Mertz (2009), for example, list the following factors as contributing to why fewer females than males across several countries excel in mathematics at the high and highest levels:



[D]ynamics in school classrooms leading teachers to provide more attention to boys; guidance counselors, biased by stereotypes, advising females against taking engineering courses; mathematically gifted girls not being identified and nurtured; scarcity of women role models in math-intensive careers leading girls to believe they do not belong in them; unconscious bias against females in hiring decisions; and hostile work environments leading qualified women to drop out in favor of friendlier climes. (Hyde and Mertz 2009, p. 8806)

What we do learn from converging reflections on neuroscientific findings regarding the nature of negative differences which matter significantly in conversations about equity and the proposed and implemented interventions, however, involve developing purposeful, targeted experiences, more inclusive instruction (Hinton et al. 2008), better programs that can effectively nurture (Hyde and Mertz 2009, p. 8806), and those that are able to “improve behavior regulation capabilities” despite the possibility of not being able to fully eliminate them (Ito and Bartholow 2009, p. 259). Eliot’s (2010) point about “experience” being a primary interventional tool provides a good provisional closure as follows:

Brain differences are indisputably biological, but they are not necessarily hardwired. The crucial, often overlooked fact is that experience itself changes brain structure and function. Neuroscientists call this shaping plasticity, and it is the basis of all learning and much of children’s mental development. Even something as simple as the act of seeing depends on normal visual experiences in early life, without which a baby’s visual brain fails to wire up properly and his or her vision is permanently impaired. (Eliot 2010, p. 22)

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